Fall 2013 **Math 601** Name: ____________________

Quiz 1

Question 1. (12 pts)

(a) (5 pts) Find equations of the line L that passes through the points $A(1, 0, 4, 3)$ and $B(3, 2, 0, 1)$.

Solution: First, calculate the direction of the line:

$$\overrightarrow{AB} = (2, 2, -4, -2)$$

Then the equations of the line are

$$\begin{cases} x_1 = 2t + 1 \\ x_2 = 2t \\ x_3 = -4t + 4 \\ x_4 = -2t + 3 \end{cases}$$

(b) (7 pts) Find an equation of the hyperplane H that passes through $(1, 1, 1)$ and is parallel to both vectors $v = (1, 0, 2)$ and $u = (0, 1, 3)$.

Solution: We need a normal vector of the hyperplane.

$$n = \begin{vmatrix} i & j & k \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{vmatrix} = (-2, -3, 1)$$

The an equation of hyperplane is of the form

$$-2x - 3y + z = b$$

for some $b \in \mathbb{R}$. Now plug $(1, 1, 1)$ into this equation to solve for b, and we have

$$-2x - 3y + z = -4$$

Question 2. (8 pts)

This problem provides a method to decide whether four given points in $\mathbb{R}^3$ lie in the same plane (i.e. coplanar). Given $P(1,2,1), Q(1,1,2), R(3,0,1), S(4,1,3)$.

(a) Write down the vectors $\overrightarrow{PQ}, \overrightarrow{PR}$ and $\overrightarrow{PS}$.

Solution:

$$\overrightarrow{PQ} = (0,-1,1)$$
$$\overrightarrow{PR} = (2,-2,0)$$
$$\overrightarrow{PS} = (3,-1,2)$$

(b) Decide whether $\overrightarrow{PQ}, \overrightarrow{PR}$ and $\overrightarrow{PS}$ are coplanar. (Hint: think about the volume of the parallelepiped spanned by these three vectors.)

Solution: $\overrightarrow{PQ}, \overrightarrow{PR}$ and $\overrightarrow{PS}$ are coplanar if and only if the volume of the parallelepiped spanned by them is zero.

The volume is

$$|\overrightarrow{PQ} \cdot (\overrightarrow{PR} \times \overrightarrow{PS})| = \cdots = 8$$

Here I leave out the details, which you can fill in yourself.

So $\overrightarrow{PQ}, \overrightarrow{PR}$ and $\overrightarrow{PS}$ are not coplanar.

(c) Conclude that whether P, Q, R and S are coplanar or not, based on part (b).

Solution: It follows from part (b) that P, Q, R and S are not coplanar.